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E2 STATIC MOMENTS AND E2, E4 TRANSITION MOMENTS
BY COULOMB EXCITATION

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ABSTRACT

Coulomb excitation, when done at low enough bombarding energies so that nuclear interactions do not interfere, is a useful method for determining nuclear electric moments. With heavy ions, multiple excitation processes predominate, allowing the determination of the moments of more states; but so many levels may be involved, that the calculations become quite complicated. Since a semi-classical theory is usually employed, quantal corrections must also be considered. Nevertheless, accuracy of a few percent in $B(E2)$ values can be obtained. Static quadrupole moments for the first 2^+ state in many even-even nuclei have been measured, although some ambiguities remain. Recently a number of $E4$ moments have been determined, and they appear to be quite large, $\sim 100 B(E4)_{sp}$.

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INTRODUCTION

Coulomb excitation is the process in which two colliding nuclei excite each other via the rapidly changing electromagnetic field between them. If the nuclei do not approach close enough to allow the nuclear forces to act, the process is, in principle, completely understood. Certain nuclear moments can be determined from the excitation yields, namely the electric moments, and indirectly, M1 moments. In particular, this method has contributed greatly to our knowledge of ground-band quadrupole moments.

Reviews¹⁻³ on the theory of Coulomb excitation should be consulted for mathematical details, but the factors governing the (classical) excitation process can be seen from the first figure. Passage of the projectile (assumed a point charge so that only monopole-multipole interactions are considered) produces a torque on the deformed target nucleus via the latter's multipole moments. If the time of passage of the projectile by the target $\tau_c \sim \frac{2\pi a}{v}$ is short compared to the characteristic time of the mode of nuclear excitation, $\tau_{ex} \sim \frac{2\pi\hbar}{\Delta E}$ that is, if $\frac{a\Delta E}{\hbar v} = \xi \ll 1$, the target nucleus feels an impulse and after the collision begins to rotate (or vibrate) in the excited state. But if $\xi \gg 1$, the target nucleus

turns during the collision, following the movement of the projectile adiabatically, and results in a vanishing probability of excitation. With a given projectile-target system, a decrease in the value of ξ can only be achieved by increasing the bombarding energy, but such an increase conflicts with the classical condition that $\eta = \frac{Z_1 Z_2 e^2}{\hbar v} \gg 1$, namely that the Coulomb interaction keeps the nuclei well apart. The resolution of this conflict is important in setting the conditions for an actual Coulomb excitation experiment and should always be kept in mind. It is also clear from Fig. 1, at least qualitatively, that the probability of Coulomb excitation is greater, the larger the electric moment of the target and the larger the charge on the projectile, as well as the higher the velocity between them. Finally, the condition $\eta \gg 1$ and the further assumption that the energy of the excited state is small compared to the kinetic energy of the projectile allows approximating the particle orbit as a classical hyperbolic trajectory.

For studies involving protons as projectiles the probability of excitation is small, and only one or two states are usually excited. But with heavy ions of large nuclear charge, excitation probabilities may become large, so that in a single collision several quanta may be exchanged. In such multiple excitation of

the target, several states, including those of considerably different spin, may be excited. The process is complicated, as a particular state may be made directly, or as the result of several excitations and de-excitations, and so its probability may no longer increase with bombarding energy, but may show an oscillating behavior.²

EXPERIMENTAL METHODS AND ANALYSIS

The experimental yields of Coulomb-excited states can be obtained by two general techniques: 1) detection of the scattered particles and determination of the inelastic yield with respect to the elastic cross section; 2) determination of the yield of de-exciting gamma rays (conversion electrons), sometimes in coincidence with scattered particles. The first method is the more straightforward one, and has the advantages that each projectile group corresponds to the excitation of a definite level and that the ratio of the yield to that of Rutherford scattering is a direct measure of the cross section for that level. A difficulty is that separation of the inelastic peaks from the more intense elastic one may require very high energy resolution, taxing the limits of present-day solid-state detectors, or more usually, magnetic spectrometers. In addition, high resolution requires beams of good energy homogeneity and thin targets, and the latter requirement

may become a severe limitation with very heavy projectiles. For example, for backward scattering a 10 mg/cm^2 target of Sm will contribute ~ 4 keV for a 10 MeV ^4He beam, 160 keV for a 100 MeV ^{40}Ar beam, and 570 keV for a 330 MeV ^{132}Xe beam. Also, because the cross sections for elastic scattering are so large, even small amounts of impurities in the target may give rise to interfering peaks in the spectrum.

The detection of the de-exciting gamma rays following Coulomb excitation offers the advantages of the very good energy resolution of Ge detectors (~ 2 keV) and, because of the possibility of using thicker targets, higher counting rates. Furthermore, these features are not affected when using very heavy projectiles (with the exception of the possibility of Doppler broadening of the peaks of very fast transitions because of the initially large velocity of the recoiling nuclei). In addition, the beam need not have very good energy homogeneity, at least to the point where the energy spread corresponds to a measurable change in the excitation cross section. However, a disadvantage is that there is no simple way to normalize the yields, as for example to Rutherford scattering with the particle detection method, and so a number of calibration corrections must be made. These include corrections for detector efficiency and geometry, target thickness, and beam stopping powers. The beam current must be determined, as also must the gamma-ray

angular distribution. If the nucleus recoils into vacuum from a relatively thin target, then the deorientation of the gamma-ray angular distribution by the hyperfine field of the unpaired electrons acting on the nuclear magnetic moment must also be determined.⁴ In addition, the modes of decay of the excited level, i.e., branched or cascade gamma-ray decay, conversion electrons, etc., must be known or determined in order to obtain absolute yields from gamma-ray measurements. An important and common variant on the gamma-ray technique, especially for multiple excitation, is to observe the de-exciting gamma-ray cascade in coincidence with backward-scattered particles. The advantage of this method is that in multiple excitation the back-scattered particle has the highest probability of exciting the target nucleus, and scaling these particles allows normalization to Rutherford scattering.

A problem common to all methods of detecting Coulomb excitation is the compromise necessary between the conditions $\eta \gg 1$ and $\xi < 1$ mentioned earlier. Since the excitation probability rises strongly with bombarding energy, it is desirable to work as near the Coulomb barrier as possible, but one must then show that interference with nuclear inelastic scattering is not occurring. A number of such studies have been performed,⁵⁻¹³ usually comparing the ratio of elastic or inelastic scattering near 180° to Rutherford. The resulting empirical rules for estimating the maximum safe

bombarding energy have the form

$$E_{\max} = \frac{1.44Z_1Z_2}{r_0 \left(A_1^{1/3} + A_2^{1/3} \right) + t \left(1 + \frac{A_1}{A_2} \right)} \quad (1)$$

where r_0 equals 1.2 or 1.25 fm and t is chosen between 5 and 6 fm.

This corresponds to lower energies than suggested as safe previously.⁵

Once the Coulomb excitation measurement has been performed and the experimental yields have been determined, how then are the moments extracted from these data? If only a single-step excitation is involved (a high-lying energy level excited by a light projectile, so that the excitation probability is small), the reduced transition moment, $B(E\lambda)$, is directly proportional to the cross section and may be evaluated employing the semi-classical or quantum mechanical expressions given in ref. 1. For multiple excitations, it is necessary to use higher-order perturbation expressions, or more usually the deBoer-Winther computer program.¹⁴ This code, a semi-classical calculation, solves the set of coupled differential equations in the time-dependent amplitudes of a finite number of nuclear states reached during the collision. The nuclear matrix elements connecting all the levels being considered must be used as input data, and then the program computes the excitation probabilities and cross section and the angular distributions of

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the de-exciting gamma rays. If the values of all the necessary matrix elements but the one(s) of interest are known from other experiments or can be estimated accurately enough, the remaining one(s) can be determined from a comparison of the calculated and experimental yields. Clearly if one uses incorrect values of the other matrix elements, or leaves out important ones (for example, elements connecting the states of interest with higher-lying levels such as β - or γ - vibrational bands or octupole bands, or elements of higher multipole moments), or is not aware of other processes leading to population of the states of interest (for example, feeding by gamma decay from higher-lying levels, or by virtual excitation through intermediate states, such as E1 polarization of the target, or by simultaneous excitation of the projectile and target), or is not justified in using the semi-classical theory, the moment determined will be in error.

RESULTS

E2 Moments

One of the earliest systematic results of Coulomb excitation by p and α beams (single excitation) was to indicate that the first excited 2^+ state in even-even nuclei of the rare-earth region showed a minimum energy when the number of neutrons and protons was between magic numbers, and that these low-lying states had very large quadrupole transition moments connecting them to the ground state. This was beautiful confirmation of the collective

rotational model of Bohr and Mottelson.¹⁵ In recent years, as heavier ions have become available as projectiles, multiple Coulomb excitation processes have become possible, and one type of study has dealt with the excitation of higher spin states in the ground band. The determination of the $B(E2)$ values in this band makes it possible to study deviations from the rigid-rotor model. That is, although the energy level spacings of well-deformed even-even nuclei in the rare-earth and actinide regions do conform generally to the expression for the rotor model, $E_I = AI(I+1)$, it is well known that, in detail, they deviate from this simple expression by a small amount which gets larger, the higher the spin, and the further the nucleus is from the center of the well-deformed region. It is of interest to determine whether the $B(E2)$ values for the ground bands, and higher bands, of such nuclei conform to the predictions of a similarly simple expression of the rotor model,

$$B(E2; I \rightarrow I-2) = B(E2; 2 \rightarrow 0) \frac{\langle I020 | I-20 \rangle^2}{\langle 2020 | 00 \rangle^2}, \quad (2)$$

or also show deviations. Determining this is far more difficult than measuring the energies, for whereas it is easy to measure gamma-ray energies to a few tenths percent, one must work quite hard to make Coulomb excitation yield measurements to a few percent. Since the deviations are expected to increase with spin as some

power of the latter, it is helpful to measure the $B(E2)$ values to as high a spin as possible. One way to indicate the deviations in the ground-band transition moments is by means of the parameter α ,

$$B(E2; I \rightarrow I-2) = B(E2; 2 \rightarrow 0) \frac{\langle 1020 | I-20 \rangle^2}{\langle 2020 | 00 \rangle^2} \times \left\{ \frac{1 + \frac{\alpha}{2} [I(I+1) + (I-2)(I-1)]}{1 + 3\alpha} \right\}^2 \quad (3)$$

Thus, for example, a study of multiple Coulomb excitation of the ground-state bands in ^{166}Er and $^{172,174,176}\text{Yb}$ to the 8^+ level was performed with ^4He and ^{16}O beams, using gamma-ray detection in coincidence with back-scattered projectiles.¹⁶ Results were calculated by means of the deBoer-Winther computer program, and the values of α derived from the $B(E2)$'s measured are shown in Table 1. On a simple model it is also possible to relate α to the energy-level deviations by $\alpha = -B/A$, where $E_I = AI(I+1) + BI^2(I+1)^2$. The values of $-B/A$ determined from the 0^+ , 2^+ , 4^+ states are also shown in Table 1. The small positive values of the order of $(0.5 \pm 0.1) \times 10^{-3}$ indicate that these nuclei are all good rotors. But the $B(E2)$ values lead to larger negative values of α , namely $\langle \alpha \rangle \sim (-1.4 \pm 1.0) \times 10^{-3}$. This would indicate centrifugal shrinking of these nuclei with increasing spin, and would suggest

that different processes are responsible for the energy level deviations than for the changes in transition moments. The resolution of this problem appears to lie in the use of the semi-classical theory; although $\eta \sim 50$ for the ^{16}O experiments, the quantal corrections for the higher spin states produced by multiple excitation are not negligible. The large value of η only indicates that this correction for a single-step excitation, the $0^+ \rightarrow 2^+$ transition, is small. No exact corrections are available for these particular third and fourth order cases, but estimates made from cases given in the literature indicate that the quantal corrections will increase the $B(E2)$ values enough to correspond to $\Delta\alpha = +1 \times 10^{-3}$ and thus to bring $\langle\alpha\rangle$ for these nuclei to essentially zero.^{17,18} To the several percent accuracy of these experiments, the transition moments for these nuclei appear to follow the expression for a rigid rotor. But clearly, to obtain more accurate E2 transition moments, better quantal corrections¹⁹ will be just as important as better experimental yields.

Much larger (positive) values of $-B/A$ are given by ^{150}Nd , ^{152}Sm , and ^{154}Gd , the 90-neutron nuclei that mark the onset of deformation as one leaves the region of the 82-neutron magic number. In these nuclei the β -vibration band also lies quite low (~ 680 keV), so that appreciable β -band mixing into the ground band (centrifugal stretching) might occur. Several groups have measured ground-band $B(E2)$ values in ^{152}Sm by multiple Coulomb excitation and the results

are given in Table 2 in terms of an average α , $\langle \alpha \rangle$. Only similar ^{16}O experiments involving gamma-ray detection in coincidence with back-scattered particles are included,^{16,20,21} and it can be seen that there is general agreement, but with some variation in the values obtained by the different groups. It is not clear whether these differences are experimental or because the various groups have treated the corrections differently and have used a different number of states in the calculation. All groups mentioned that the quantal corrections were an unknown, but hopefully small, quantity; we still do not know the exact value, but it now seems that it is not so small and is equivalent to about an increase of 1 in the first three values of α listed in Table 2. The states we have included in our calculation²¹ are shown in Fig. 2; those whose energies are listed in parentheses are estimated. The effects of the various corrections made in our calculation on the reduced transition moments are shown in Table 3; on the whole they are small, but not negligible. It can be seen that the calculation of these quadrupole transition moments in the ground band of ^{152}Sm is not a completely straightforward procedure if accuracies of the order of a few percent are desired. Such accuracy is probably possible if the effects of all important higher bands (both from multiple excitation and from decay after excitation), higher multipole moments in the ground band, quantal corrections, and El

polarization are included.

E4 Moments.

The realization that the E4 moment was important came because we obtained different values of $\langle \alpha \rangle$ when the ^{152}Sm target was Coulomb excited with other projectiles besides ^{16}O , namely ^4He and ^{40}Ar . The value of α for ^{40}Ar excitation was smaller, $\langle \alpha \rangle = (+0.7 \pm 1.1) \times 10^{-3}$, while that for ^4He was much larger.²¹ We do not yet know the reason for the difference between ^{16}O and ^{40}Ar , but the $^4\text{He} - ^{16}\text{O}$ difference was so large as to indicate that some physical feature had been left out of the calculation. To try to decide which of the answers was correct, if either, an independent determination of the ^{152}Sm ground-band $B(E2)$ values was performed using the recoil-distance Doppler-shift technique.²² Coulomb excitation is used to excite the levels, but Coulomb excitation theory and its corrections are not used directly. The value of α obtained from this study was $\langle \alpha \rangle = (1.9 \pm 0.6) \times 10^{-3}$ as listed in Table 2. This is closer to the ^{16}O value than to the much larger ^4He value, and says something was wrong with the ^4He calculation. We concluded that direct excitation of the 4^+ state via the E4 moment of ^{152}Sm must be considered.

The effect of E4 single excitation is more important with ^4He projectiles (and even more so with protons) than with heavier ions because the 2nd order E2 excitation is not so overwhelmingly large

compared to the weak $E4$ process. Thus it should be possible to measure the reduced $E4$ transition moment, $B(E4; 0 \rightarrow 4)$, by a careful measurement of the Coulomb excitation yield of the 4^+ state with ${}^4\text{He}$ particles, if the $B(E2; 0 \rightarrow 2)$ and $B(E2; 2 \rightarrow 4)$ are known; the ratio of the experimental yield of the 4^+ state to that calculated from the $B(E2)$ values alone can be compared with ratios calculated for different values of $\langle 0^+ || M(E4) || 4^+ \rangle$. In Fig. 3 is shown a plot of such ratios for 10.4 MeV ${}^4\text{He}$ ions back-scattered on ${}^{152}\text{Sm}$ (ref. 23). The largest source of error comes from the two $B(E2)$ values. It can be seen that a small error in them makes a proportionately much larger change in the $E4$ matrix element, as the total increase in cross section from that with $B(E4) = 0$ is only 11% in this example. The major correction to the calculated cross section comes from the quantal corrections; for ${}^{152}\text{Sm}$ excited by ~ 10 MeV ${}^4\text{He}$ this amounts to a 1% reduction in the calculated 2^+ cross section and a 7% reduction in the 4^+ one, but these result in almost a 30% increase in the reduced $E4$ matrix element for ${}^{152}\text{Sm}$ (ref. 24).

A number of hexadecapole moments have been measured by this technique in the last two years,²³⁻²⁹ and a partial listing of the reduced matrix elements is given in Table 4. The values are quite large, corresponding to reduced transition probabilities of over 100 $B(E4)_{\text{sp}}$. Values of β_2 and β_4 derived from these $B(E4)$ values and the corresponding $B(E2; 0 \rightarrow 2)$ ones by model dependent calculations

are in general agreement with theoretical values for the rare-earth region.³⁰ Four separate groups have measured $^{152,154}\text{Sm}$; the values for ^{152}Sm agree, within the stated errors, and the values for ^{154}Sm do not quite agree. The main difference in the answers comes from the choice of $B(E2)$ values used.

Static Quadrupole Moments.

A special case of multiple Coulomb excitation is the process known as nuclear reorientation.^{5,31-33} It appears in perturbation theory as an interference between the first- and second-order processes connecting the ground state and the excited state, and makes the yield of the latter slightly dependent upon the signs and magnitudes of the quadrupole moments of the ground and excited states. Figure 4a shows this situation for an even-even nucleus where only the 2^+ excited state has a static quadrupole moment, and illustrates how the name originated. In addition to the reorientation process, the 2^+ state can be reached by (virtual) excitation through other states, of which a higher 2^+ state and the giant dipole state are shown as examples in Fig. 4b. The yield of the 2^+ state must be corrected for these additional modes in order to determine that due to the reorientation process itself, and the sign of the matrix elements connecting these states can introduce an ambiguity into the answer. The perturbation expression for the ratio of the reorientation effect to the first order process has

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the form

$$\frac{P_{i \rightarrow f}^{(12)}}{P_{i \rightarrow f}^{(11)}} \approx \frac{A_1}{Z_2} \frac{\Delta E}{1+A_1/A_2} \langle 2^+ \| \eta(E2) \| 2^+ \rangle K(\xi, \theta) \quad (4)$$

where subscript 1 refers to the projectile, subscript 2 to the target, ΔE is the excitation energy, $\langle 2^+ \| \eta(E2) \| 2^+ \rangle$ is the reduced static E2 matrix element of the 2^+ state of interest, and $K(\xi, \theta)$ is a function which varies relatively slowly with ξ but decreases quite markedly from $\theta = \pi$ to $\theta = 0$. The measurement can be done by determining the 2^+ yield as a function of the projectile atomic number or of the projectile scattering angle, but in either case the experiment requires considerable care as the effect is only of the order of 10% with ^{16}O beams. Clearly, there is an advantage in going to heavier projectiles.

One of the earliest and most exciting reorientation experiments was the determination of the static quadrupole moment of the first 2^+ state in ^{114}Cd . The initial results gave a surprisingly large value, almost equal to the rigid-rotor value derived from the reduced transition probability, $^{34-36} B(E2; 0 \rightarrow 2)$, although the energy level spacings of such a nucleus suggest a harmonic vibrator. A truly harmonic vibrator, however, has a zero quadrupole moment, and it was difficult for theoreticians to calculate the large value

observed. Considerable interest was stimulated, and most of the possible reorientation methods were used to measure this nucleus.^{11, 38-42} The results are listed chronologically in Table 5; the values found range from 0 to $0.85eb^2$. Clearly, some of the experiments must be in error. For the first three experiments involving yields determined from (particle-coincident) gamma-ray intensities,³⁴⁻³⁶ the attenuation of the gamma-ray angular distribution from the Cd nuclei recoiling out of the target into a vacuum⁴ had been neglected. As learned later, this attenuation depends on the recoil velocity, and hence on the projectile mass (at a given particle velocity). In the detector configuration used this leads to too large a reduction of the gamma-ray intensity with heavier ions, hence too large a value for Q_{2+} . This effect has now been studied with ^{16}O beams, and the gamma-ray experiment has been repeated, giving a lower value.⁴¹ Experiments comparing the intensities of the inelastic and elastic peaks have given still lower values, but also the high value, and it is not so easy to see the origin of the differences here. However, the Q_{2+} determined in this way does depend upon the value of $B(E2; 0 \rightarrow 2)$ which is used or simultaneously determined, and there is a 10% spread in these numbers or just about enough to help explain the Q_{2+} discrepancy. Measurement of the $B(E2)$ by an independent method

It may also be seen in the same figure that the Ba nuclei with fewer than 82 neutrons show an increasing static quadrupole moment as the number of neutron holes is increased, that is, an increase in the polarization of the core also as one moves below the magic number. But the different sets of measurements give different signs, ^{36,48,49} an even more startling development than with the

^{114}Cd results, where only the magnitude was in question. This situation certainly calls for a careful reinvestigation of these systems.

The final set of even-even nuclei to be mentioned are Pt and Os. Experiments by the Pittsburgh group⁵⁰⁻⁵² have given quadrupole moments (Fig. 6) that clearly go from large negative values in the light Os nuclei to a large positive value for ^{198}Pt . These data provide perhaps the most direct evidence that a transition from prolate to oblate shapes has occurred in these nuclei. A different type of reorientation measurement, that of the precession of the γ -ray angular distribution,^{52a} has also indicated the oblate nature of ^{194}Pt .

No mention has been made of light nuclei in this talk, nor of odd-mass nuclei, but not because they have not been studied. Reorientation studies in light nuclei have been extensively pursued, and bring in the two new features of projectile reorientation and reorientation measurements using analysis of the Doppler-broadened gamma-ray lineshapes,^{53,54} but this subject will be covered in the following talk by Dr. Häusser. In addition, interpretation of quadrupole moments of light nuclei, both static and dynamic, will be given in a later presentation by Dr. Cline. Odd-mass nuclei have been omitted purely because of time limitations.

Finally, I should mention the future use of still heavier ions

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in Coulomb excitation. There is an interesting contribution¹³ to this session in which a low-energy (0.7 MeV/nuc) ^{81}Br beam was used to do a reorientation experiment on ^{170}Er ; the high Z of the projectile tends to magnify the reorientation effect, and the low energy guarantees no nuclear penetration and greatly reduces the effects of higher-lying states. The result is a good compromise to the conflicting requirements mentioned at the beginning that $\eta \gg 1$ but $\xi < 1$; if a variety of ions are available, the compromise can perhaps be optimized for a particular type of experiment. But with still heavier ions of energies capable of reaching near the Coulomb barrier, the whole range of multiple excitation studies will be enlarged greatly, Doppler-shift studies will become easier and more important, and new processes, such as Coulomb-excited fission, may become possible. The last figure shows the calculated excitation probabilities for the ground-state rotational band of ^{238}U with a number of heavy projectiles; the dashed line at 1% probability indicates a probability which can conveniently be seen with present techniques for ions up to ^{40}Ar . Using this criterion, the 34^+ state should be observable by the use of Fb on ^{238}U . Just the observation of such high-spin states as unique members of the ground band will be a very interesting development for present-day ideas on the nature of the yrast levels⁵⁵ and on the origin of the

"back-bending" observed in ground-band energy-level spacing in certain nuclei.⁵⁶ But to determine quantitative data from such measurements, e.g. $B(E2)$ values, will require much larger or more efficiently arranged computer codes than now exist in order to take into account the effects of a great number of states, and will necessitate an experimental program of carefully determining a great number of $B(E2)$ values with a series of smaller projectiles, as the necessary input data will have to be built up, matrix element by matrix element. In addition, it would appear that accurate results from multiple excitation studies, even some of those already performed with moderate-sized projectiles, may have to await the calculation of good high-order quantal corrections. But, the prospects for Coulomb excitation studies in the future are very exciting.

FOOTNOTES AND REFERENCES

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Table 1. Values of $\langle \alpha \rangle$ from B(E2) and Energy Level Measurements.¹⁶

Nucleus	$10^3 \langle \alpha \rangle$ from B(E2)	$-10^3 B/A$ from 0,2,4 levels
^{166}Er	$-(2.1 \pm 1.0)$	0.9 ± 0.2
^{172}Yb	$-(1.3 \pm 0.9)$	0.6 ± 0.1
^{174}Yb	$-(0.5 \pm 1.0)$	0.5 ± 0.1
^{176}Yb	$-(1.6 \pm 1.0)$	0.5 ± 0.1

Table 2. Average values of α for ^{152}Sm .

Method	$10^3 \langle \alpha \rangle$
Multiple Excit., ref. 20	3.4 ± 1.0
Multiple Excit., ref. 16	2.7 ± 1.7
Multiple Excit., ref. 21	1.7 ± 1.4
-B/A from 0,2,4 levels	6.7 ± 1
Plunger, ref. 22	1.9 ± 0.6

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Table 3. Correction (in Percent) to Calculated $B(E2; I \rightarrow I-2)$ of ^{152}Sm from 54 MeV ^{16}O Scattered at 160° (ref. 21).

Cause	I = 4	I = 6	I = 8
β -band	-1.0	+1.4	+0.6
γ -band	+1.5	+1.8	+1.8
K=0 octupole band	+0.4	+3.6	+8.3
K=1 octupole band	+0.9	-1.2	-3.6
$E4$ moment	+3.7	+10.5	+19.6
Total	+5.5	+16.1	+26.7
$10^3 \Delta\alpha$	+3.7	+3.2	+2.6

Table 4. Reduced Hexadecapole Matrix Elements

Nucleus	Reference	$\langle 4^+ \eta(E4) 0^+ \rangle$ in eb^2
^{152}Sm	23	0.45 ± 0.09
	27	0.35 ± 0.07
	28	0.47 ± 0.07
	29	0.37 ± 0.09
^{154}Sm	24	0.67 ± 0.08
	27	0.43 ± 0.08
	28	0.65 ± 0.05
	29	0.54 ± 0.11
^{158}Gd	27	0.39 ± 0.11
	28	0.39 ± 0.09
^{160}Gd	27	0.36 ± 0.10
^{162}Dy	27	0.27 ± 0.10
^{164}Dy	27	0.28 ± 0.11
	28	0.25 ± 0.16
^{166}Er	27	$0.06^{+0.12}_{-0.18}$
	28	0.12 ± 0.18
^{168}Er	27	$0.20^{+0.12}_{-0.18}$
	28	0.12 ± 0.20
^{170}Er	27	$0.24^{+0.14}_{-0.18}$

(continued)

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Table 4 (continued)

Nucleus	Reference	$\langle 4^+ \eta(E4) 0^+ \rangle$ in eb^2
$^{174}_{Yb}$	28	0.23 ± 0.17
$^{230}_{Th}$	25	1.04 ± 0.21
$^{232}_{Th}$	26	1.60 ± 0.27
$^{234}_{U}$	26	1.70 ± 0.10
$^{236}_{U}$	26	1.23 ± 0.28
$^{238}_{U}$	25	1.12 ± 0.23
$^{238}_{Pu}$	26	1.45 ± 0.21
$^{240}_{Pu}$	26	1.18 ± 0.22
$^{242}_{Pu}$	26	$0.70^{+0.3}_{-0.4}$
$^{244}_{Pu}$	26	$0.0^{+0.7}_{-0.6}$
$^{244}_{Cm}$	26	-0.4 ± 0.4
$^{246}_{Cm}$	26	-0.4 ± 0.4
$^{248}_{Cm}$	26	$0.0^{+0.4}_{-0.6}$

Table 5. Quadrupole Moments of ^{114}Cd .

Reference	$B(E2; 0 \rightarrow 2)$ in $e^2 b^2$	Q_{2+} in eb
34	0.51 ± 0.02	-0.70 ± 0.21
34a		-0.6 ± 0.2
35	0.561 ± 0.005	-0.85 ± 0.15
36	0.48 ± 0.05	-0.49 ± 0.25
37	0.509 ± 0.009	$+0.05 \pm 0.27$
38	0.561 ± 0.017	-0.68 ± 0.09
39		-0.64 ± 0.19
40		-0.53 ± 0.17
41	0.498 ± 0.027	-0.40 ± 0.12
11	0.513 ± 0.005	-0.28 ± 0.09
42		-0.35 ± 0.07

FIGURE CAPTIONS

Fig. 1. Projectile of atomic number Z_1 Coulombically scattered along classical hyperbolic orbit by target nucleus Z_2 . Torque exerted on latter through its electric moments causes excitation (rotation) of the target.

Fig. 2. Schematic view of recoil-distance Doppler-shift method used for measuring ground-band lifetimes of ^{152}Sm (ref. 22).

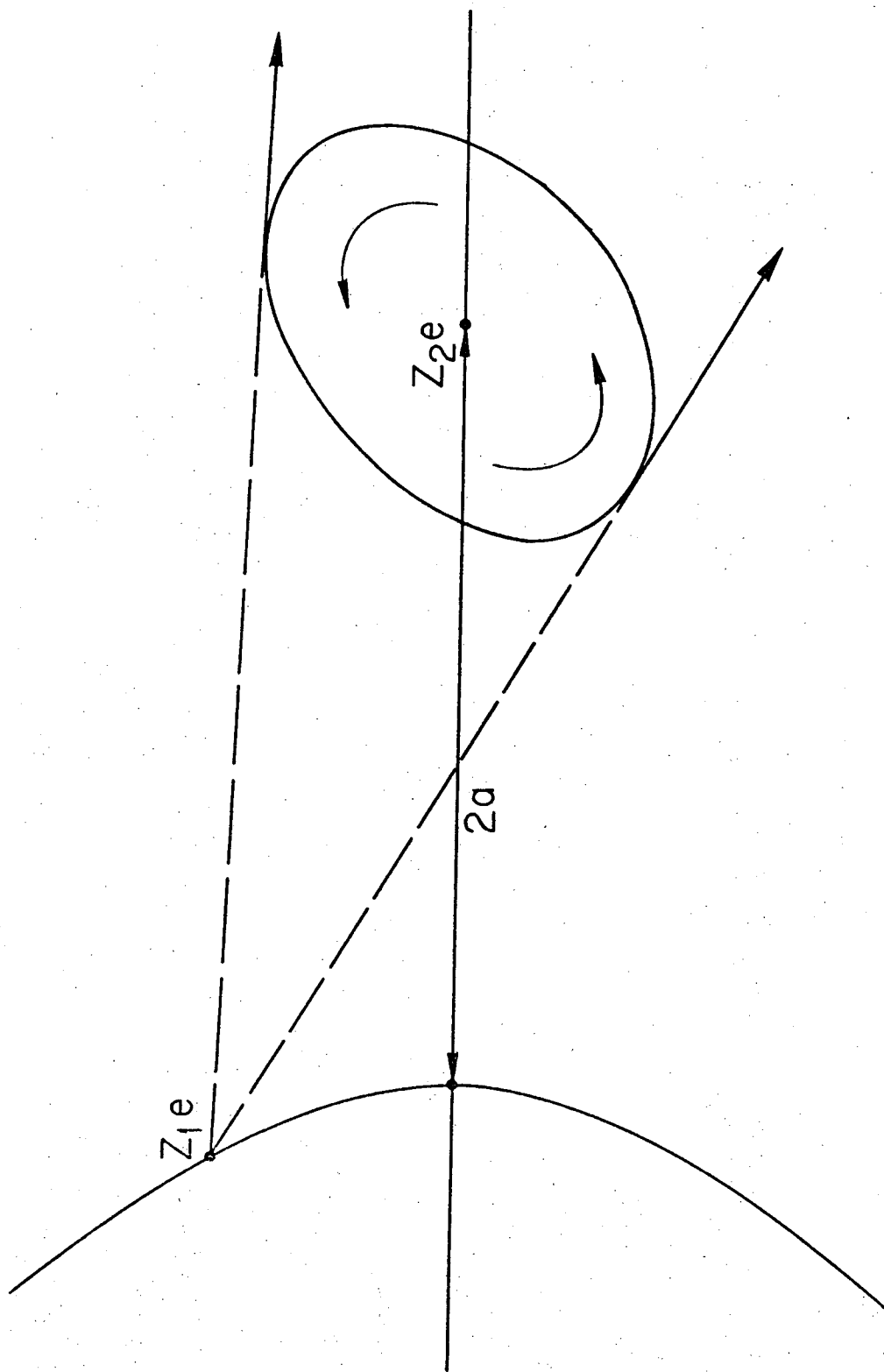
Fig. 3. Relationship between reduced $E4$ matrix element and a) the back-scatter cross section for 10.4 MeV ^4He ions normalized to the case of zero $E4$ matrix element and b) the deformation parameter, β_4 , using a radius of $R_0 = 1.2 A^{1/3}$ fm and a β_2 which yields the experimental $E2$ moment (ref. 23).

Fig. 4. a) Illustration of interference between 1st order excitation of $2+$ state and 2nd order excitation, from which "reorientation process" obtains name. b) Schematic of 1st order excitation, reorientation effect, interference with higher-lying $2+$ state, and interference by higher-lying $1+$ state (for example, virtual excitation through the giant dipole state).

Fig. 5. Plot of Q_{2+} for Ba (refs. 36, 48, 49), Nd (refs. 44, 45) and Sm (refs. 36, 46, 46a, 47, 47a) nuclei around neutron number 82.

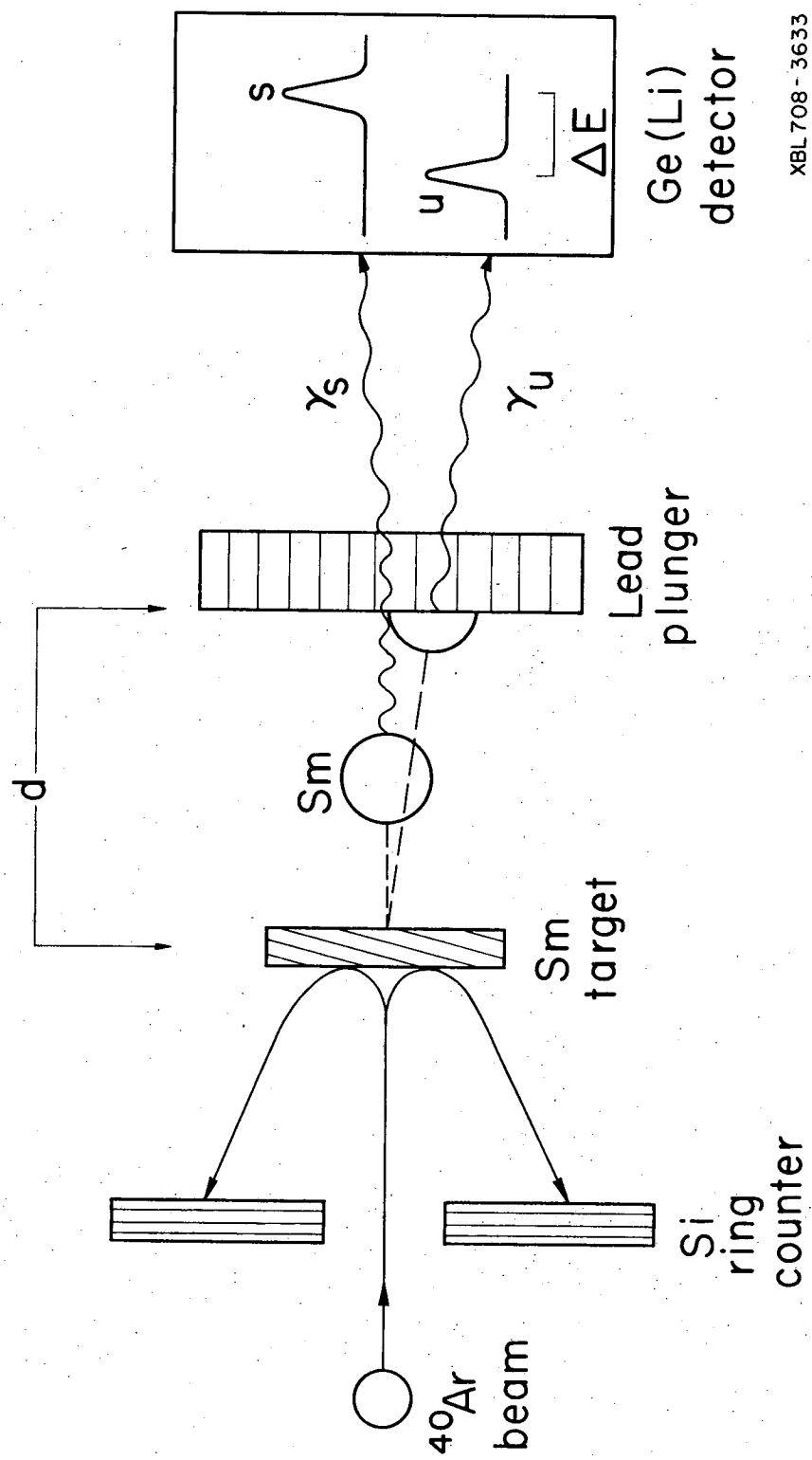
Fig. 6. Plot of Q_{2+} for Os (ref. 50, 51) and Pt (ref. 52) nuclei vs. neutron number.

Fig. 7. Probability of exciting the level of spin I in ^{238}U vs. I for a number of heavy ions, as calculated by the deBoer-Winther Coulomb excitation program, using only ground-band levels connected by rigid rotor matrix elements.



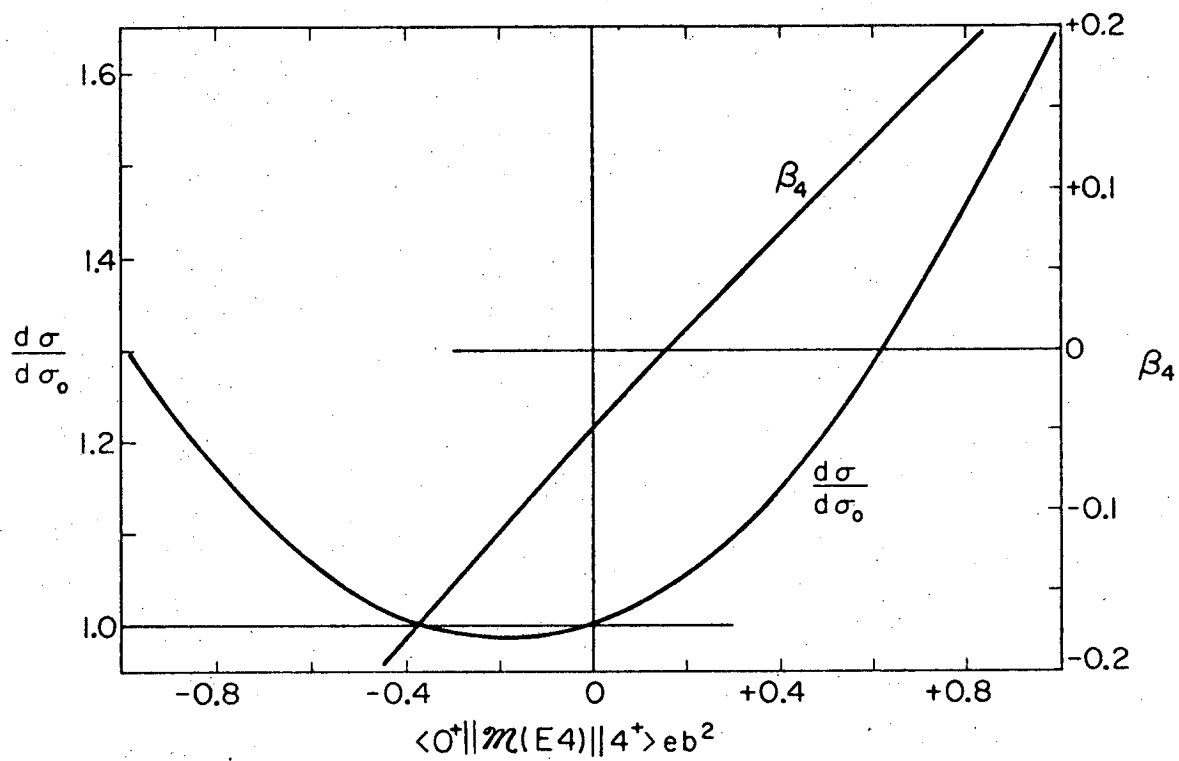
XBL 728-3764

Fig. 1



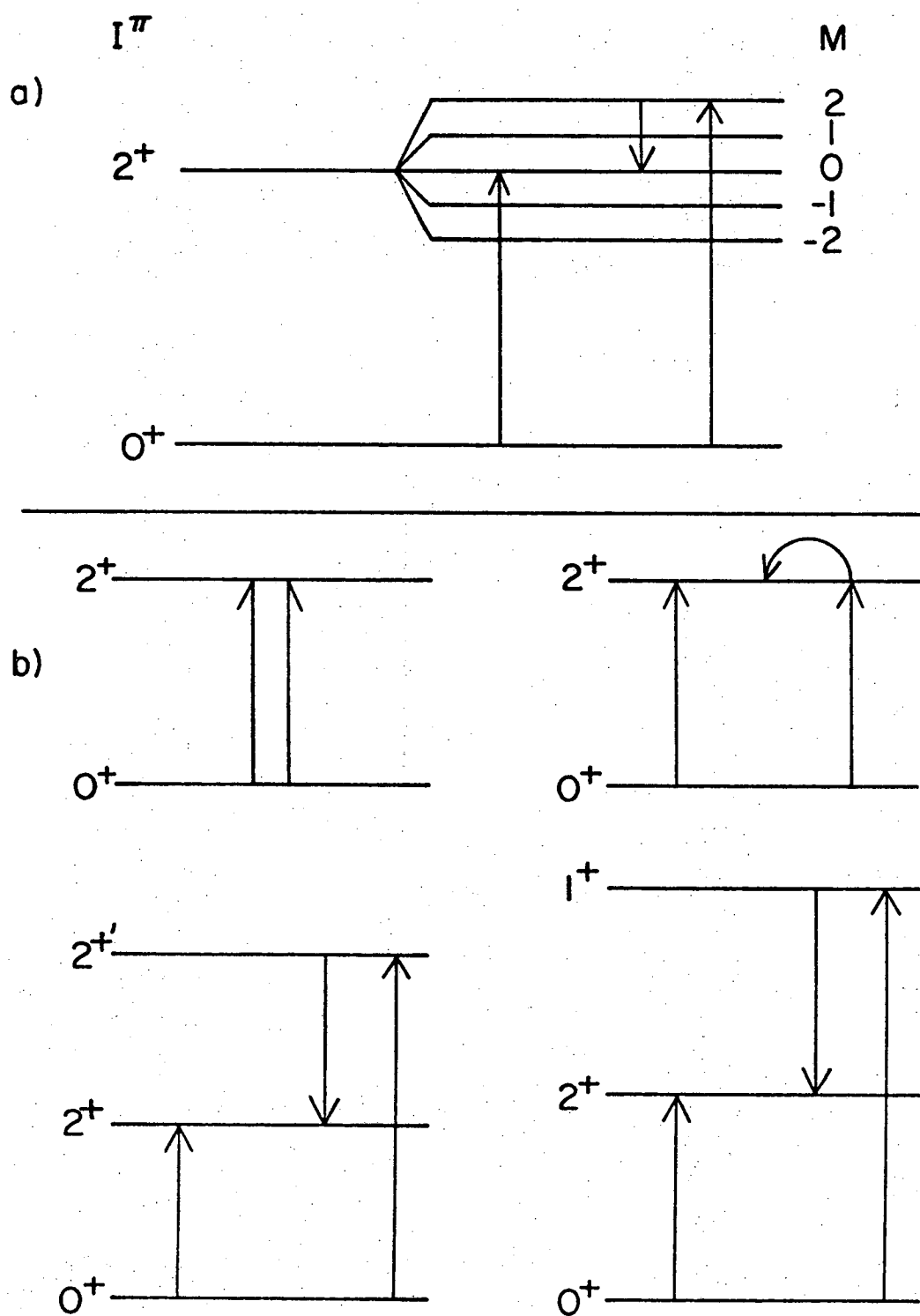
XBL 708 - 3633

Fig. 2



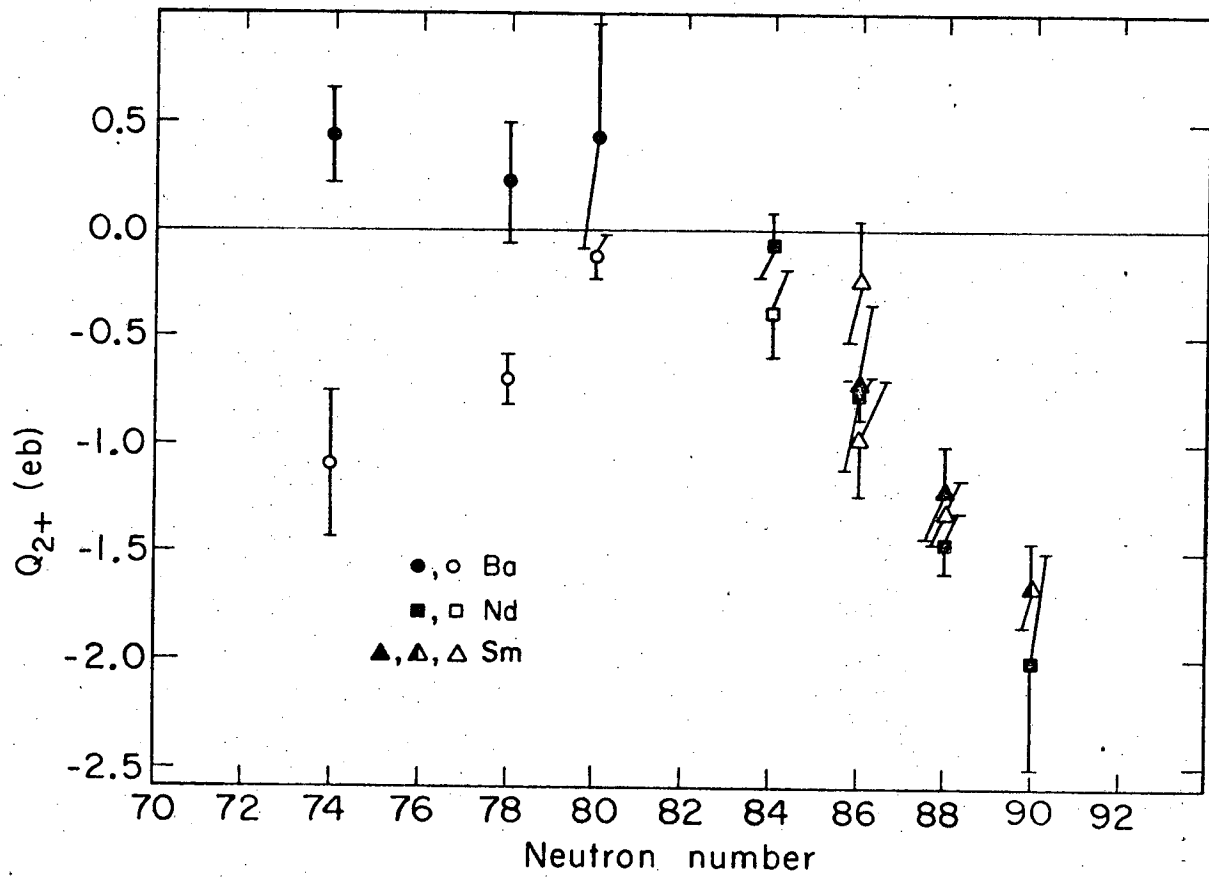
XBL703-2496

Fig. 3



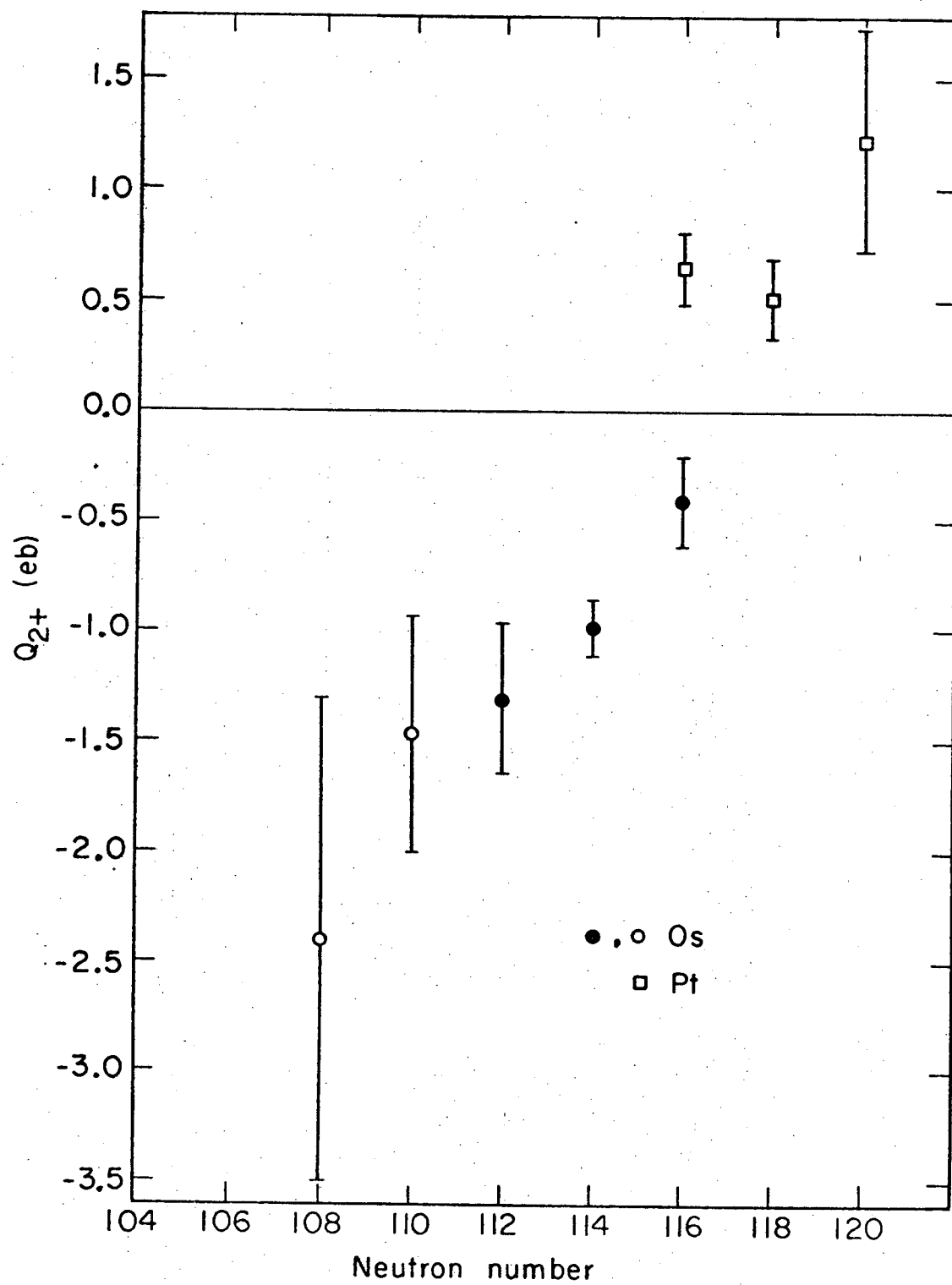
XBL728 - 3765

Fig. 4



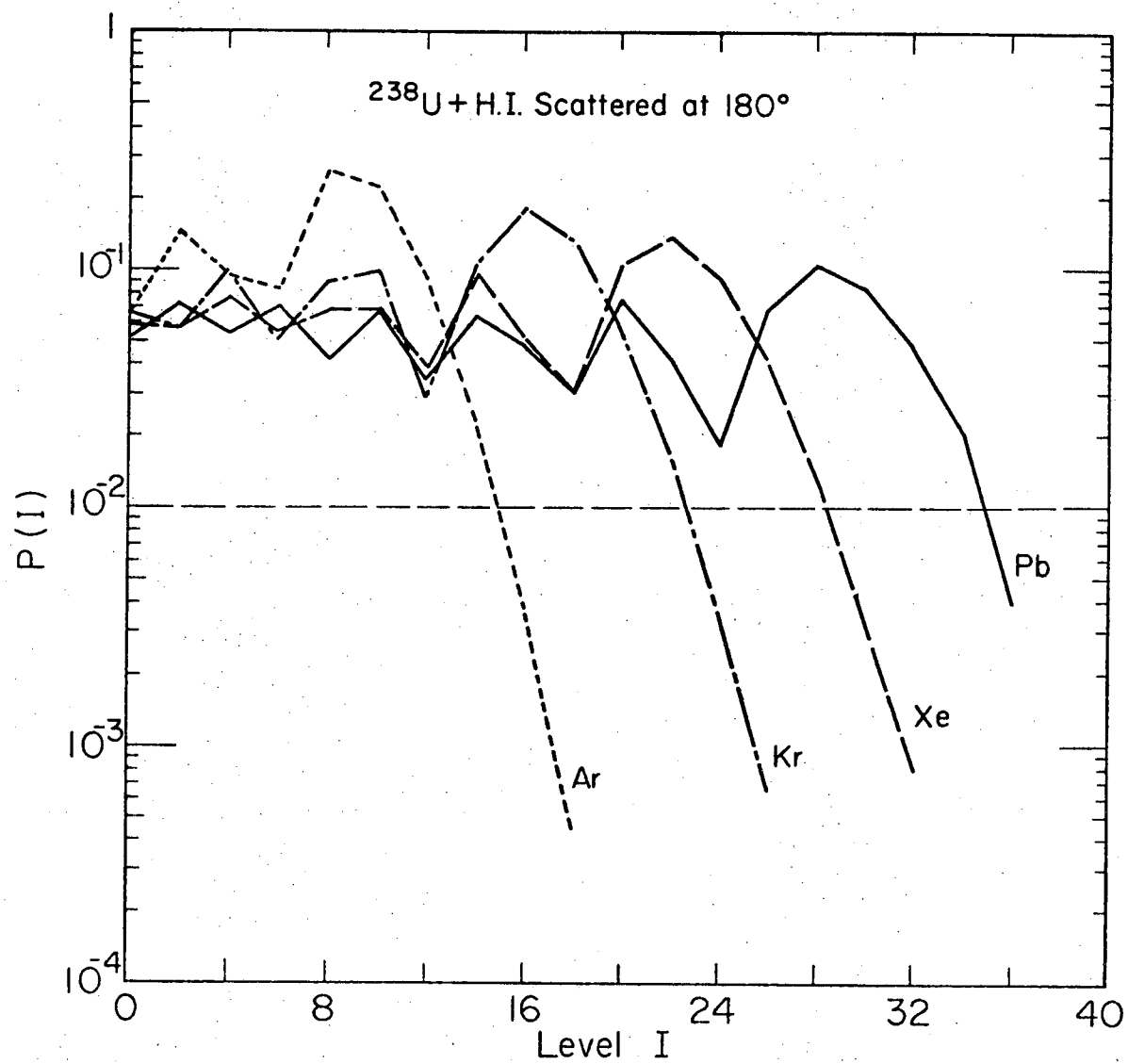
XBL728-3766

Fig. 5



XBL728-3767

Fig. 6



XBL728-3763

Fig. 7

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